Understanding Variables

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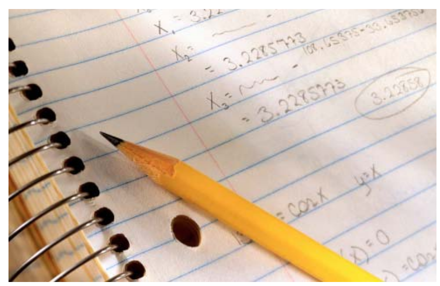
Summary: Describes how to add, subject, multiply, and divide variables.
Learning Objectives: To define the terms variable, coefficient, and term. To add, subtract, multiply, and divide variables.

Variables are placeholders for some unknown number. Our first introduction to variables was in kindergarten when we were given number strings like "1, 2, __, 4, 5, __, 7, 8, 9, 10." The blanks represented unknown numbers we were asked to fill in. Later, as we learned to add and subtract or multiply and divide, we saw a box or a circle for the unknown number that we were supposed to find. Eventually, variables became letters (*a, b, x, y*). Let's now review how to identify a variable and how to use it in an algebraic expression.



The most common variables are *x*, *y*, and *z*, but a variable can be ANY letter, shape, or character that is used in place of an unknown number, sometimes called a quantity. Variables are often assigned special letters depending on what we are looking for. For instance, in a formula where we are looking for the mass of an object when we are given the density and volume of the object, the variable would likely be *m*, for mass. The letter *m* is a reminder that we are looking for the mass.

Variables are often accompanied by a number. The number in front of a variable is called the **coefficient**. This coefficient tells you how many of that variable you have. For example, in *10x*, *10* is the coefficient – you have 10 *x*'s. In cases where you have a variable (*x*) without a coefficient, it is understood that you have a coefficient of 1 (you have only one *x*). This is often referred to as the "invisible coefficient."

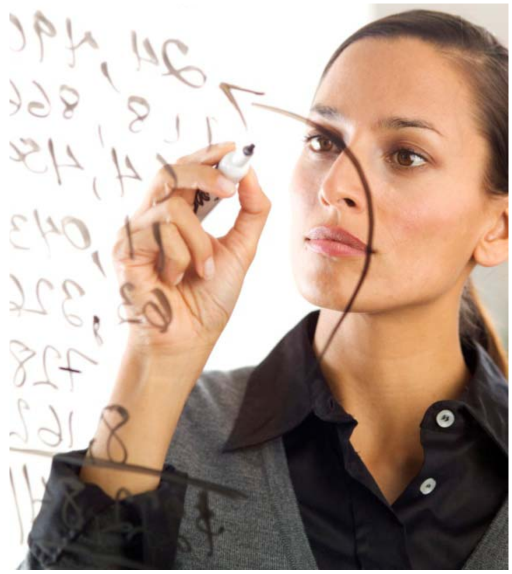
A coefficient and a variable together (like 10x or 4r) are called a **term**. Terms can have more than one variable. For example 7yz and $8h^2k$ are also terms.

Variables follow the same rules (known as the Order of Operations) that numbers do in equations or expressions. However, variables do have some further restrictions within those rules. This handout does not review the Order of Operations, but will discuss rules regarding variables for addition, subtraction, multiplication, and division. But, if you need a refresher on the Order of Operations, you can find a handout that reviews the rules at www.uhv.edu/ac/mathsci.

Adding and Subtracting Variables

Adding and subtracting variables can only be done when the variables are exactly alike. If the variables differ in any way, including exponents, then they cannot be added or subtracted. Adding and subtracting variables is called **combining like terms**.

Let's consider two examples.



The expression 2x + 4y – 7x + 3y can be rewritten as (2x – 7x) + (4y + 3y) and then combined to get -5x + 7y. Notice that we put all the *x*'s together and all the *y*'s together. We are allowed to do this because of the Commutative Property of Addition that says we can move addition terms around without changing values. Since we had more negative *x*'s than positive *x's* our answer was negative.

The expression $19xy - 4x + 12xy^2$ cannot be combined, however, since there are no like terms. Each of the terms is different, that is *xy* is not the same as either *x* or xy^2, and these last two terms are not the same as each other. When the terms cannot be combined any further, they are said to be in the **simplest form**.

Multiplying and Dividing Variables

When multiplying variables you put the variables together, and multiply any coefficients. If the variables are different letters, like 5x · 7y, they will combine (*xy*, in this case), and the coefficients are multiplied (to become *35*). The variables and coefficients then become a single term (*35xy* in this case). As a general rule, the variables are put into alphabetical order when combined.

When you have two terms with the same letter variables you will add the **exponents** of the **variables**. You will do this even if the exponents are different. While this would make them different **terms** for adding and subtracting, in multiplying (and dividing) you can put them together.

For example $5x \cdot 7x = (5{\cdot}7)(x^{1+1})35x^2$

$10xy^3 \cdot 14x^6y^2 = (10 \cdot 40)(x^{1+6}y^{3+2}) = 140x^7y^5$

Like that "invisible coefficient," if there is no exponent indicated on a variable, it is understood to be 1.

Dividing variables works the same as multiplying them. You will divide the coefficients and combine the variables. When they are the same letter variable you will subtract the exponents of the variable in the denominator from the exponent of the variable in the numerator.

For example $64x^3y / 2xy = (64/2)(x^{3-1}y^{1-1}) = 32x^2$

(Here we are able to divide the coefficients and the variables. Since there is one *y* in the numerator and one *y* in the denominator, they cancel each other out.)

$15xy / 3z = (15/3)(xy/z) = 5xy / z$

(In this case only the coefficients could be divided because all the variables were different.)

When you cannot evenly divide the coefficients, then you must get them to their simplest fraction form.

For example $64x^3y / 12xy = (64/12)(x^{3-1}y^{1-1}) = 16x^2/3$

If the variable in the denominator has a larger exponent than its counterpart in the numerator, you will still subtract the exponents, but keep the variable in the denominator.
For example: $75x^2y^5 / 15x^3 = (75/15)(x^{2-3}y^{5-0}) = 5y^5/x$

For more on multiplying and dividing exponents see the *Multiplying and Dividing Exponents* handout at www.uhv.edu/ac/mathsci.

Practice Exercises

Try these exercises for some practice.

Add/Subtract the like terms

1. $14x – 7y + 8x + 3y^2$

2. $12w^4 – 5w^3 – w^4$

3. $8x + 3x – 2x$

4. $2x – 5 + 4y + 3x -7y + 4$

5. $3xy – 5x +8 – y – x^2y$

Multiply or Divide

6. $15xy \cdot 7yz$

7. $(4y^2)(8x^2y)$

8. $42a^2 \div 18a^3$

9. $325yz \div 25yz^2$

Answers:

Add/Subtract the like terms

1. $14x - 7y + 8x + 3y^2 \rightarrow (14x + 8x) - 7y + 3y^2 \rightarrow 22x - 7y + 3y^2$ (y and y^3 are NOT like terms)
2. $12w^4 - 5w^3 - w^4 \rightarrow (12w^4 - 1w^4) - 5w^3 \rightarrow 11w^4 - 5w^3$
3. $8x + 3x - 2x \rightarrow 9x$ (Since they are all like terms, the expression can be simplified down to only one term)
4. $2x - 5 + 4y + 3x - 7y + 4 \rightarrow (2x + 3x) + (4x - 7y) + (-5 + 4) \rightarrow 5x + (-3y) + 9 - 1) \rightarrow 5x - 3y - 1$
5. $3xy - 5x + 8 - y - x^2y \rightarrow$ simplest form already

Multiply or Divide

6. $15xy \cdot 7yz \rightarrow (15 \cdot 7)(x \cdot y^{(1+1)} \cdot z \rightarrow 105xy^2z$
7. $(4y^2)(8x^2y) \rightarrow (4 \cdot 8)(x^2 \cdot y^{(2+1)}) \rightarrow 32x^2y3$
8. $42a^2 \div 18a^3 \rightarrow (\frac{42}{18})(a^{(2-3)}) \rightarrow \frac{7}{3a}$
9. $325yz \div 25yz^2 \rightarrow (\frac{325}{25})(y^{(1-1)} \cdot z^{(4-2)}) \rightarrow 13z^2$

As always, if you feel you would like more help regarding the topic covered in this handout and you are a UHV student, you are welcome to meet with the Academic Center tutors for a face-to-face session. Check www.uhv.edu/ac/tutoring/subject.aspx for current tutor availability.