

## Understanding Angles

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**Summary:** Learn to the basic terminology and geometry of angles.

**Learning Objectives:** To define angles. To state and define the classifications of angles. To describe how angles are measured. To determine whether angles are positive or negative.

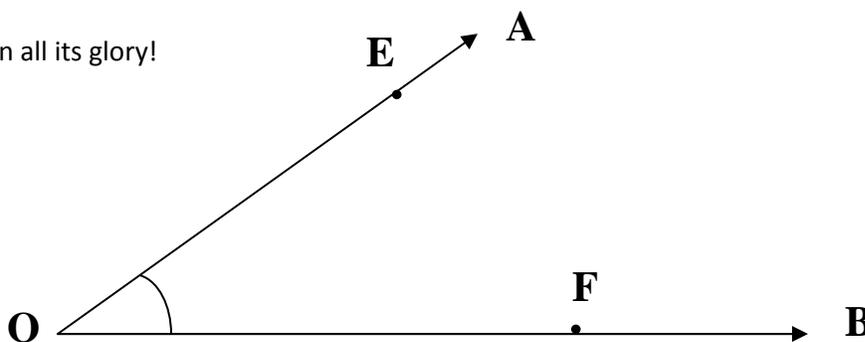
Angles are everywhere. This sheet of paper has four right angles (the corners), each measuring 90 degrees. Understanding angles is important for many kinds of career paths, such as teaching, aviation, architecture, graphic arts, geology, carpentry, engineering, cartography, and many more. Are you ready for a crash course in angles?

After you read this handout, you will know how to define angles, how angles are measured, and what the standard classifications of angles are (acute, right, obtuse, straight, and reflex). Finally, this handout will discuss the standard positions of angles.

### What is an Angle?

An angle is a figure formed by two lines or rays meeting at a common endpoint, called a vertex.

Behold the angle in all its glory!



Let's examine the terminology of an angle and the way that angles and lines within them are generally written.

The **vertex** is the point where two lines meet. In this case the vertex is labeled **O** (which generally refers to "origin").

The **rays** in this figure are  $\overrightarrow{OA}$  and  $\overrightarrow{OB}$ , which are the same as  $\vec{A}$  and  $\vec{B}$  respectively. The little arrow over the letters is a math symbol that denotes a ray. Note: these are sometimes called **arms** of the angle.



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The **lines** in this figure are  $\overline{OE}$  and  $\overline{OF}$ . The little lines over the letters denote that these are line segments.

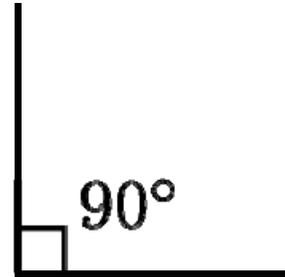
We denote angles with the symbol  $\angle$ , thus the **angle** in the figure is  $\angle O$ . You can also label the angle by including the letters designating the rays as  $\angle AOB$ ,  $\angle BOA$ ,  $\angle EOF$ , or  $\angle FOE$ .

The little arc in the diagram is a common way of denoting angles.

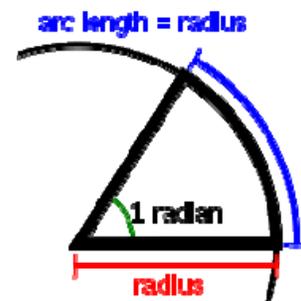
### How Are Angles Measured?

The common units of angular measure are degrees and radians.

A **degree** (or arcdegree) is a measurement of an angle representing  $1/360$  of a full rotation, such as the circumference of a circle, and is denoted by a small superscript circle ( $^\circ$ ). The figure  $90^\circ$  is read as "ninety degrees."



A **radian** is the standard unit of angular measure representing an arc length that is equal to the radius. Radian measurements can be abbreviated to *rad*. However, radians are frequently assumed unless otherwise specified, so *rad* may be omitted in some texts.



A complete rotation (or revolution) about the origin is  $360^\circ$ . Likewise, a complete rotation about the origin is  $2\pi$  radians.

$$360^\circ = 2\pi \text{ rad}$$

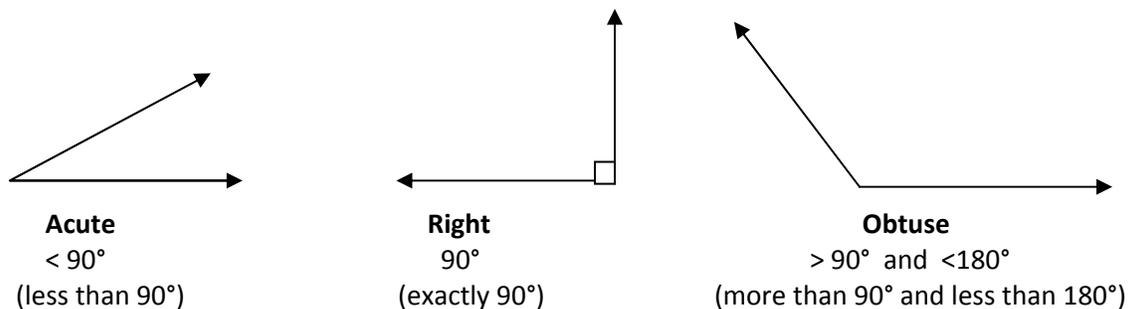
This chart below lists the common angle measurements in both degrees and radians.

Units	Values							
Degrees	$0^\circ$	$30^\circ$	$45^\circ$	$60^\circ$	$90^\circ$	$180^\circ$	$270^\circ$	$360^\circ$
Radians	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\pi$	$\frac{3\pi}{2}$	$2\pi$

### How can Angles be Classified?

We can classify angles by their measurement as acute, right, obtuse, straight, and reflex.

**Acute** angles have less than  $90^\circ$ , **right** angles have exactly  $90^\circ$ , and **obtuse** angles have more than  $90^\circ$  but less than  $180^\circ$ . All three are illustrated below.

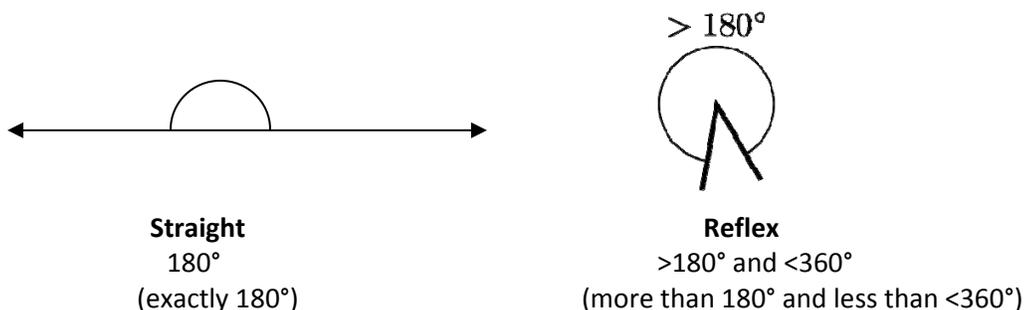


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**Note: Be careful! The less than sign (<) and the angle sign (∠) are two distinct symbols.**

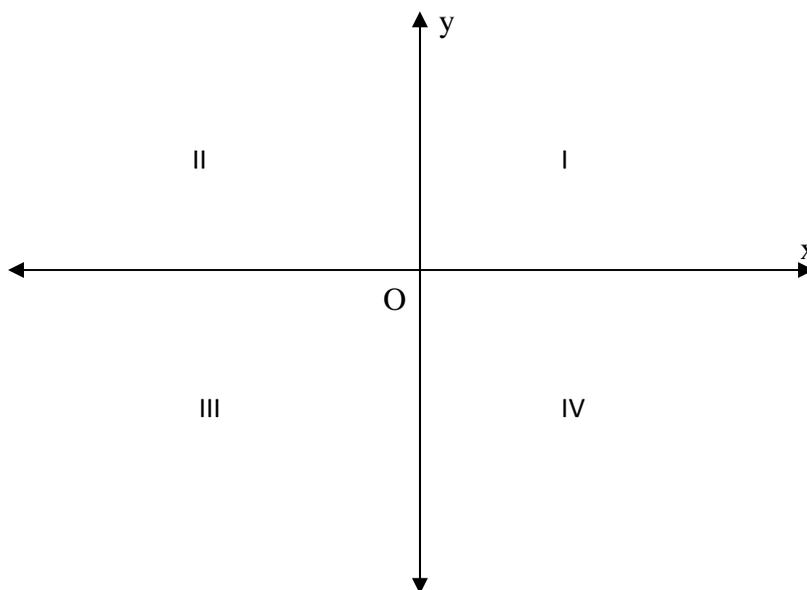
**Straight** angles have exactly  $180^\circ$  (and look like straight lines), and **reflex** angles have more than  $180^\circ$  but less than  $360^\circ$ . The arcs in the following diagrams help distinguish which angles are being measured.



### Standard Position of Angles

In order to understand the standard position of angles, it's important to understand the Cartesian plane, also known as the xy-plane.

The Cartesian plane is a grid comprised of a horizontal x-axis and a vertical y-axis that intersect at the origin and divide the plane into four quadrants, I, II, III, and IV.



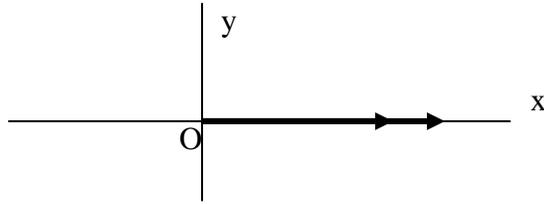
A **standard position angle** is an angle on the Cartesian plane whose vertex is at the origin and one ray is on the positive x-axis (to the right of the origin). The arm of the angle that sits on the x-axis is called the **initial side**, and the other arm is called the **terminal side**.



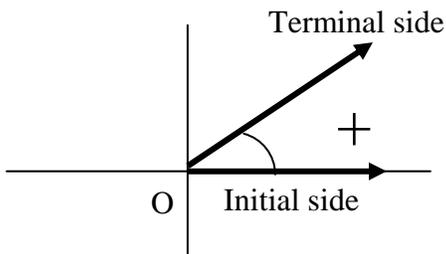
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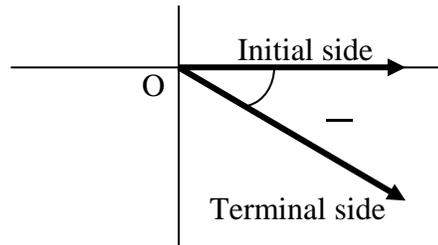
The next figure shows 2 rays extending positively on the x-axis, representing an angular measure of  $0^\circ$ .



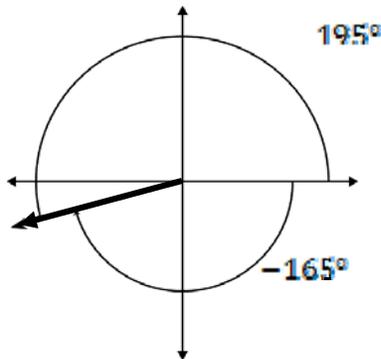
When you raise one ray upward in the counter-clockwise direction, this creates a **positive** angle.



When you lower one ray downward in the clockwise direction, this creates a **negative** angle.



When 2 angles in standard position have the same terminal side, they are said to be **coterminal angles**, such as in the picture below.



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### Practice Exercises

In exercises 1 through 6, the measurements of angles are given. Label the following angles as **acute**, **right**, **obtuse**, **straight**, or **reflex**.

1.  $179.99^\circ$     2.  $\frac{\pi}{2}$     3.  $45^\circ$     4.  $\frac{4\pi}{3}$     5.  $300^\circ$     6.  $2^\circ$

7. On the Cartesian plane, is the y-axis horizontal or vertical?
8. Angles that are in the standard position always have one arm that sits on the positive x-axis, and it is called the \_\_\_\_\_ side.
9. When the terminal side of a standard position angle rotates counter-clockwise, the angle measurement is \_\_\_\_\_.
10. When the terminal side of a standard position angle rotates clockwise, the angle measurement is \_\_\_\_\_.

### Answers

1. Obtuse    2. Right    3. Acute    4. Reflex    5. Reflex    6. Acute
7. Vertical    8. Initial Side    9. Positive    10. Negative

For further understanding of angles, you may want to look at these other handouts:

- More on Angles: Vertical, Corresponding, and Alternate Interior and Exterior Angles
- More on Angles: Complementary and Supplementary Angles
- Radians and Degrees



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