# Factoring Numbers

**Author/Creation:** Pamela Dorr, September 2010.
**Summary:** Describes two methods to help students determine the factors of a number.
**Learning Objectives:** To define prime number and composite number. To factor prime and composite numbers. To factor the prime numbers of any composite number. To use the factor tree and cake methods to factor numbers. To recite the divisibility rules.

Factoring numbers means that we break numbers down into the other whole numbers that multiply together to make our beginning number. That is to say that if we are looking for the factors of *a* then we will look for all the numbers that multiply together to make *a.* For instance, 2 and 6 are factors of 12 (2 x 6 = 12), so are 3 and 4 (3 x 4 = 12), and 1 and 12 (1 x 12 = 12).

In the next few sections we will

1. Define prime number and composite number;
2. Discuss how to determine the factors of composite numbers; and
3. Discuss how to determine prime factors using two different methods.

### Basic Definitions

Before we begin to talk about factoring numbers it is important to understand that there are two types of numbers. There are *prime numbers* and *composite numbers*. What's the difference?

A **prime number** does not have any factors other than 1 and itself. The numbers 2, 3, 5, 7, 9, and 11 are prime numbers because their only factors are 1 and themselves. They are also the first few prime numbers and are used to make up the majority of the composite numbers.

A **composite number** has at least three factors: 1, itself, and any other numbers that multiply together to make the composite number. Examples of composite numbers are 6, 8, and 10. Each of these numbers has more than just 1 and itself as factors. For instance 6 has the factors 1, 2, 3, and 6. That is 1 x 6 = 6 and 2 x 3 = 6.

A **factor** is a number that can be multiplied by another number to get a third number. In our example above, 2 and 3 are both factors of 6 because they can be multiplied together to get 6. The numbers 1 and 6 are also factors of 6 since they can be multiplied together to get 6.

## Factoring

To **factor a number**, you will look for all the numbers that can be multiplied together to make up your target number. For some more common numbers this process is fairly easy. With our earlier example, we see that 1, 2, 3, and 6 are all factors of 6, but how did we get there? Since 1 is a factor for all numbers, we will start with 2. Does 2 divide evenly into 6? Yes, it does, so it is a factor. When we divide 6 by 2, our quotient is 3, so 3 is another factor. Then you move on to 3. In this case we see that we have already found 3 to be a factor of 6, so we can stop factoring.

But what about a larger number? How can we find the factors of 36 or 85? There are some basic divisibility rules that we can use to find factors of numbers. Let's look at these now. Remember also, that once you find one factor of a number, if you divide that factor into your original number, your quotient will be a second factor.

2 – All even numbers are divisible by 2 (So for 36, 2 is a factor since 2 x 18 = 36. This means that 18 is also a factor of 36. For 85, 2 is not a factor since 2 does not divide evenly into 85.) Remember all even numbers end in a 2, 4, 6, 8, or 0.

3 – Find the sum of all digits in the number. If the sum is divisible by 3, then the number is too. (For 36: 3 + 6 = 9. Since 9 is divisible by 3, so is 36. For 85: 8 +5 = 13. Since 13 is not divisible by 3, neither is 85.)

4 – If the last two digits of a number are divisible by 4 then, the number is divisible by 4. (For 36: we know that 36 is divisible by 4 since 4 x 9 =36. For 85: since 85 ÷ 4 = 21 ¼, 4 is not a factor of 85.)

5 – If the last digit is a 5 or a 0, then the number is divisible by 5. (Since 85 ends with a 5, it is divisible by 5; 5 x 17 = 85. Since 36 does not end with a 5 or 0, it is not divisible by 5.)

6 – If the number is divisible by BOTH 2 and 3, then it is divisible by 6. (Since 36 is divisible by BOTH 2 and 3, it is divisible by 6. Since 85 is not divisible by both 2 and 3, it is not divisible by 6.)

7 – Double the last digit and subtract it from the rest of the number. If your answer is 0 or divisible by 7, then the number is divisible by 7. (For example: 672 – Double the 2 (2 x 2 = 4) and subtract that from 67 (67 – 4 = 63). Since 63 is divisible by 7 (7 x 9 = 63) then 672 is divisible by 7. By contrast 905 is not. Double the 5 (5 x 2 = 10) then subtract the product from 90 (90 – 10 = 80). Since 80 is not divisible by 7, then 905 is not divisible by 7. Our numbers of 36 and 85 are not divisible by 7 either.)

8 – If the last 3 digits of a number are divisible by 8, then the number itself is divisible by 8. (For example: 5816 is divisible by 8 since 816 ÷ 8 = 102 and 4836 is not since 836 ÷ 8 is 104 ½ . Our other examples of 36 and 85 are not divisible by 8 either.)

9 – Find the sum of all digits in the number and if it is divisible by 9, then the number is divisible by 9. (For example 36: 3 + 6 = 9 Since 9 is divisible by 9, 36 is divisible by 9. With 85: 8 + 5 = 13 which is not divisible by 9, so 85 is not divisible by 9.)

10 – The number ends in 0. (Since neither of our example numbers end in 0, neither of them is divisible by 10. Since 50 ends with a 0 it is divisible by 10, 10 x 5 = 50. Another example would be 8900. Since 8900 ends with a 0 it is divisible by 10, 10 x 890 = 8900.)

11 – Find the sum of every $2^{nd}$ digit and then subtract it from the sum of all the other digits. If the answer is 0 or multiples of 11, then the number is divisible by 11. (For example: 3729 – Add 7 and 9

(7 + 9 = 16) then subtract the sum of the rest of the digits (3 + 2 = 5) so 16 – 5 = 11, therefore 3729 is divisible by 11. With 2516 – The sum of 5 + 6 = 7 and the sum of 2 + 1 = 3. Since 7 – 3 = 4, then 2516 is not divisible by 11.)

12 – If the number is divisible by BOTH 3 and 4, then it is divisible by 12. (Since 36 is divisible by both 3 and 4 it is divisible by 12. Since 85 is not divisible by both 3 and 4 it is not divisible by 12.)

To get started on factoring,

1. Check each of above divisibility rules to see if any of these numbers divide into your number.
2. Then once you divide one factor into your number, you will obtain a second factor.
3. You can stop looking for factors when you begin to overlap numbers. For instance with 100, you have 1 x 100, 2 x 50, 4 x 25, 5 x 20, 10 x 10. Since 10 multiplies times itself, you do not have any other factors. Once you find the square root of a number (if there is one), you are done looking for factors.

To keep track of the factors you have already found, list them in the order as you find them.

Let's consider another example by looking at the number 120. We'll look at each of the above factors and apply their rules to 120 to see if they are factors of 120.

1 (This is obvious since 1 goes into all numbers) 1 x 120 = 120

2 (Because 120 is even, 2 is a factor.) 2 x 60 = 120

3 (Since 1 + 2 + 0 = 3 and 3 is divisible by 3, then 120 is divisible by 3.) 3 x 40 = 120

4 (Because 4 divides into 20, 4 also divides into 120.) 4 x 30 = 120

5 (Because 120 ends in a 0, it can be divided by 5.) 5 x 24 = 120

6 (Because 2 and 3 divide into 120, so does 6.) 6 x 20 = 120

7 (We double the 0 to get 0, and then subtract 0 from 12. 12 – 0 = 12 Since 12 is not divisible by 7, then neither is 120.)

8 (Since we would look at the last three digits of a number and 120 only has 3 digits, we just try dividing 8 into 120 and we see that it does work.) 8 x 15 = 120

9 (Since we already know that the sum of the digits is 3 [See the test for 3 above.], then we know that 9 will not divide evenly into 120.)

10 (Because the last digit is a 0, we know that 10 will divide into 120.) 10 x 12 = 120

11 (When we subtract 2 from the sum of 1 + 0 we get ( 1+ 0) -2 = 1 - 2 = -1, which means that 11 is not a factor of 120.)

12 (Our usual test says that if 3 and 4 are both factors, then 12 is a factor. We see that 120 passed both of the tests for 3 and 4 as factors. However, we also see in our test for 10, that 12 is the other factor that multiplication fact. We are now doubling back. This means that we have found all of our factors for 120.)

So our final list of factors would be 1, 2, 3, 4, 5, 6, 8, 9, 10, 12, 15, 20, 24, 30, 40, 60, and 120.

## Prime Factorization

So far, we've talked about factoring composite numbers. However, every composite number can be broken down to its **prime factorization**. Prime factorization is when a number is broken down into only its prime factors.

Let's look at the number 20. The number 20 can be broken down into 4 x 5. The next step is to find the factors for 4 and 5. The 4 can be broken down into 2 x 2. Five can only be divided into the factors 1 and 5, so by definition 5 is a prime number.

Since 2 and 5 are both prime numbers (each number can only be multiplied by itself and one), the prime factorization for 20 is 2 x 2 x 5. If the prime number is used more than once as a factor, this is noted in **exponential form**. For example: 20 = 2 x 2 x 5 or $2^2$ x 5. (See our handout on understanding exponents for more information on the exponential form.)

The prime factorization is different from factoring in that each number in the prime factorization can be used to create another factor for the target number. Using 20 again: $2^2$ x 5 = 20, but you could use $2^2$ to get 4 so 4 is also a factor of 20 (4 just isn't a *prime* factor of 20).

Here are some examples of numbers, their factors, and their prime factors:

| Number | Factors | Prime Factors |
|---|---|---|
| **36** | **36 1**<br>**18 2**<br>**12 3**<br>**9 4** | **2 x 2 x 3 x 3**<br>**or**<br>**$2^2$ x $3^2$** |
| **42** | **42 1**<br>**21 2**<br>**14 3**<br>**7 6** | **2 x 3 x 7** |

| Number | Factors | Prime Factors |
|---|---|---|
| **100** | **100 1**<br>**50 2**<br>**25 4**<br>**20 5**<br>**10**<br>**(we only put 10 once because 10 x 10 =100)** | **2 x 2 x 5 x 5**<br>**Or**<br>$2^2 \times 5^2$ |
| **88** | **88 1**<br>**44 2**<br>**22 4**<br>**11 8** | **2 x 2 x 2 x 11**<br>**Or**<br>$2^3 \times 11$ |

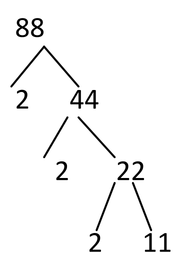
## Prime Factorization Methods

There are a few ways to find the prime factors of a number. The most commonly taught method is the factor tree.

The factor tree uses the rules of divisibility to break number down into its prime factors.

1. A factor tree starts with the original number at the top with two branches extending down.
2. At the other end of these two branches are two factors that multiply together to make up our original number. You can use either prime or composite numbers in these lower branches.
3. When you have a prime number at the end of the branch, then the branch stops there. If you have a composite number at the end of the branch, then you add two more branches off of the composite number that end with two factors of that composite number.
4. You repeat this until you have prime numbers at the end of all the branches.

Let's look at 88 again. Using the factor tree we would see:




We start with 2 here because 88 is an even number and according to our divisibility rule for 2, all even numbers are divisible by 2. We could have just as easily started with a 4 here, and then divided the 4 into two branches of 2 and 2. Then, since 44 is also even, we can use a 2 again. And finally, we see that 22 is also even, so we can once again divide by 2. We would stop here since all the branches end with a prime number.

Here we can clearly see each of the prime factors of the number 88. This method looks like a type of tree, hence the name "factor tree."

Another method to find the prime factors is the “birthday cake”, or upside down division, method. This one often looks like an upside down birthday cake when it is finished. It is, in essence, an upside down division problem. With this method, we would always divide by prime numbers.

1. We start with the original number inside our upside down division box.
2. On the outside, we put the first prime number that divides evenly into our original number. Use the divisibility rules from above to determine which factors will divide into your original number.)
3. Underneath, we put the factor that multiplies times the prime number to get our original number. This factor is our new “original number.”
4. Repeat these steps until the number underneath is also a prime factor.

Just remember that when we go through the divisibility rules with the “birthday cake” method we only use prime numbers, so the rules for 4, 6, 8, 9, 10, and 12 will not be needed.

| | |
|---|---|
| **2** | **88** |
| **2** | **44** |
| **2** | **22** |
| | **11** |

In the case of 88, the number 2 is a prime number, so we start there. We could have started with 11 since it is also a prime number. We started with 2 here because it is the first factor test we go through when we look at the divisibility rules mentioned above. Once we divide 2 into 88 we get 44 as a factor. We then go through the divisibility rules again and see that 2 also goes into 44. We repeat this process until we end up with a prime number as the next factor to check. In the end we see that $88 = 2^3 \times 11$.

## Now You Try

Try these on your own.

Decide if they are prime or composite. If they are composite, find all the factors and the prime factorization.

1. 75
2. 67
3. 94
4. 63
5. 288

Determine the prime factorization of the following numbers using the factor tree method.

1. 330
2. 18
3. 108
4. 117
5. 91

Find the prime factorization of the following numbers using the birthday cake method.

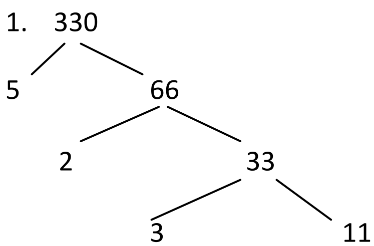
1. 85
2. 70
3. 231
4. 51
5. 625

**Answers:**

1. Composite; Factors: 1, 3, 5, 15,25,75; Prime factors: $3 \times 5^2$
2. Prime
3. Composite; Factors: 1, 2, 47, 94; Prime factors: $2 \times 47$
4. Composite; Factors: 1, 3, 7, 9, 21, 63; Prime Factors: $3^2 \times 7$
5. Composite; Factors: 1, 2, 3, 4, 6, 8, 9, 12, 16, 18, 24, 32, 46, 48, 72, 96, 144, 288; Prime Factors: $2^5 \times 3^2$
6. Prime

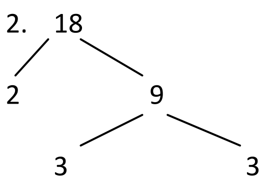
With the following factor tree problems, if you chose different beginning numbers, but the find the correct prime factorization, that is perfectly fine.

1.



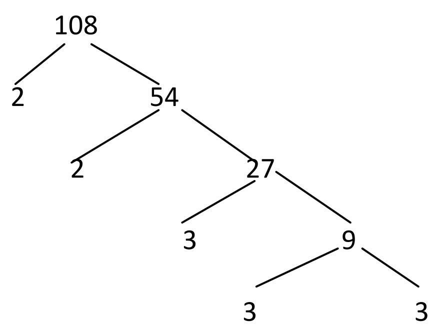

$2 \times 3 \times 5 \times 11$

2.



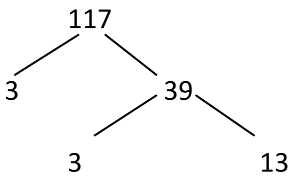

$2 \times 3^2$

3.



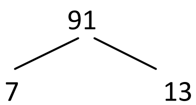

$2^2 \times 3^3$

4.




$3^2$ x 13

5.




7 x 13

With the following “birthday cake” method, it is important to only divide by prime factors, but you can start with different numbers than what are listed here.

1. 5 x 17

| | |
|---|---|
| **5** | **85** |
| | **17** |

2. 2 x 5 x 7

| | |
|---|---|
| **2** | **70** |
| **5** | **35** |
| | **7** |

3. 3 x 7 x 11

| | |
|---|---|
| **3** | **231** |
| **7** | **77** |
| | **11** |

4. 3 x 17

| | |
|---|---|
| **3** | **31** |
| | **17** |

5. $5^4$

| | |
|---|---|
| **5** | **625** |
| **5** | **125** |
| **5** | **25** |
| | **5** |

If you still have more questions about factoring numbers and are a UHV student, please come to the Academic Center for face-to-face tutoring. Academic Center hours can be found at http://www.uhv.edu/ac/tutoring/subject.aspx.