Multiplying and Dividing with Exponents

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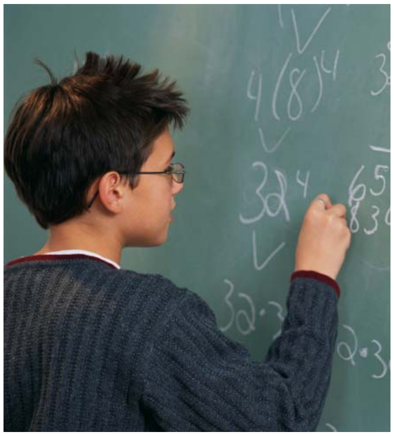
Summary: Describes how to multiply and divide numbers and variables with exponents.
Learning Objectives: To define exponent. To multiply and divide numbers and variables that have exponents, including negative and fractional exponents.

Exponents are used in many algebraic expressions and equations. Knowing how to work with exponents will be a great help in understanding how to work out algebraic problems. Exponents are a way to condense or shorten repeated multiplication of the same number.

Basic Definitions



Exponents are those little floating numbers placed above and to the right (in superscript form) of a variable or number. For example, in 7^6, 6 is the exponent and 7 is the base. The exponent indicates how many times you multiply that number or variable by itself. For example, 6^5 means 6 · 6 · 6 · 6 · 6 and x^3 means x · x · x.

The **base** is the number or variable that the exponent is attached to. In the expression 6^5, 6 is the base and 5 is the exponent. Likewise, in the expression x^3, x is the base and 3 is the exponent. In an exponential number or expression where the base is negative $(-6)^2$ the entire base (-6) is multiplied by itself the number of times indicated by the exponent -6· -6=36. (When the base is negative, an even exponent will result in an even answer; when the base is negative and the exponent is an odd number, the base will be a negative number.)

Special Exponents

A **negative exponent** means that you are looking for the **inverse.** Essentially, to form the inverse, make the base a fraction with a 1 in the numerator and the base with a positive exponent in the denominator. For example, 7^{-2} is really $1/7^2$ or 1/49 and x^{-19} is really $1/x^{19}$.

A **fractional exponent** means that you are looking for the root of the **base**, so $8^{1/3}$ means you are looking for the cubed (or third) root of 8, or 2. Another way to write it is $2^3 = 8$. Similarly, $729^{1/6}$ means you are looking for the 6^{th} root of 729, or 3. Another way to write this is 3^6=729. This means you are looking for a number that when multiplied by itself the indicated number of times will give you the original base.

Fractional exponents take the form of $a^{\frac{b}{c}}$, so when you have a number besides 1 in your numerator (the top of the fraction), you will treat this number as a normal exponent. That is, once you find the root of the base, then you multiply the base by itself the number of times indicated in the numerator of the fractional exponent. For instance, if we have $729^{\frac{3}{6}}$, then we take the 6th root of 729, or 3. Next, we multiply 3 by itself three times, or 3 x 3 x3, and we get 27. So $729^{\frac{3}{6}} = 27$.

How to Multiply Like Bases with Exponents

When you multiply like variables, or bases, with exponents you will add the exponents together, so $x^4 \cdot x^5$ would be $x^{(4+5)}$ or x^9.

To illustrate this last example, let's look at it another way: $x^4 = x{\cdot}x{\cdot}x{\cdot}x$ and $x^5 = x{\cdot}x{\cdot}x{\cdot}x{\cdot}x$. If we multiply these together we get $x{\cdot}x{\cdot}x{\cdot}x{\cdot}x{\cdot}x{\cdot}x{\cdot}x{\cdot}x$ or x^9. Another example would be $5^3 \times 5^4 = 5^{(3+4)} = 5^7$.
If the bases are different, say $x^5 \cdot y^5$, you do not add their exponents. **You can only add the exponents of like bases.**

Multiplying Exponents with Exponents

Sometimes you will see a base with its exponent raised to an exponent. This can look rather tricky. However, to solve this problem, you would multiply the two exponents together. Let's look at this example, $(a^7)^8$ would actually be the same as writing $(a^7)\cdot(a^7)\cdot(a^7)\cdot(a^7)\cdot(a^7)\cdot(a^7)\cdot(a^7)\cdot(a^7)$. Now from above we know that we would just add all the exponents up to get the final exponent. However, a quicker way to do this would be to multiply $7 \cdot 8$ and get 56. So $(a^7)^8 = a^{56}$.

How to Divide Like Bases with Exponents

When you are dividing like variables, or bases, you will subtract the exponent of the variable in the denominator from the exponent of the variable in the numerator, so x^4/x^2 becomes $x^{(4-2)}$, which then would be x^2. Let's look at this one all written out: $x^4 = x{\cdot}x{\cdot}x{\cdot}x$ and $x^2 = x{\cdot}x$, so $x^4/x^2 = \frac{x \cdot x \cdot x \cdot x}{x \cdot x}$. Since we can cancel out two x's each from the numerator and the denominator, we are left with $x \cdot x$ only, or x^2.

If the variable in the denominator has a higher exponent than its counterpart in the numerator, then you will still subtract and keep the variable in the denominator, so x^4/x^6 becomes $x^{(4-6)}$ which is x^{-2} or $1/x^2$. Remember from above that a negative exponent means you are looking for the inverse of the number. To see this one written out we have x^4 -= $x{\cdot}x{\cdot}x{\cdot}x$ and $x^6 = x{\cdot}x{\cdot}x{\cdot}x{\cdot}x{\cdot}x$, so we get $x^4/x^6 = \frac{x \cdot x \cdot x \cdot x}{x \cdot x \cdot x \cdot x \cdot x \cdot x}$. Again we cancel x's from the numerator and denominator until you are left with $\frac{1}{x \cdot x}$ or $1/x^2$.

If the bases are different, say x^8/k^3, you cannot subtract the exponents. **You can only subtract the exponents of like bases.**

Be Aware of These Special Exponent Cases

When you have the number **ten (10) as a base**, you can easily do the required exponential multiplication by putting zeros (0) behind the one (1). For instance with 10^8, you would make sure there are 8 zeros behind the one, so it would look like 100,000,000. If you have 10^{-4}, you would see that first it is a negative exponent so you are going to be looking for $1/10^4$ or 1/10,000.

Anytime you have an **exponent of 1**, your base will be your answer. Any base to the first power is itself. So $825^1 = 825$ and $825^{-1} = 1/825$.

Anytime you have an **exponent of 0**, your answer will be 1. So $9{,}897^0 = 1$. The exception to this rule is 0^0. In this case, the number is undefined.

Exponents in Algebraic Expressions

In algebra, you will often see binomial expressions with exponents. While we will not go into a full discussion of how to solve these expressions here (see the Academic Center handout *Binomial Expansion Using Pascal's Triangle* for more information on solving these expressions), we will give a brief description of how to work them.

Like other exponential expressions we have been working with here, binomials are set the same way. In the expression $(6x + 8y)^7$, *(6x + 8y)* is the base and *7* is the exponent. You would, in essence multiply (6x +8y) by itself 7 times, or $(6x + 8y)\cdot (6x +8y)\cdot (6x +8y)\cdot (6x +8y)\cdot (6x +8y)\cdot (6x +8y)\cdot (6x +8y)$. (The handout mentioned above discusses a quick and painless way to solve this expression since the larger the exponent the more confusing the multiplication can get.)

For a binomial expression that is only squared, meaning it has an exponent of 2, you can simply use the FOIL method to solve the expression (see the Academic Center handout *Solve by Factoring* for an explanation of the FOIL method).

A quick example of squaring a binomial is
$(x + y)^2 = (x + y)\cdot(x + y)= x\cdot x + x\cdot y + y\cdot x + (x + y)\cdot(x + y)= x\cdot x + x\cdot y + y\cdot x + y\cdot y = x^2 + 2xy + y^2$

It is important to note in the example above that $x\cdot y$ and $y\cdot x$ are the same thing. That is, they are equal to each other by the Commutative Property of Multiplication and can be written as either *xy* or *yx* without changing the value.

Test Your Understanding

To test your understanding, try these exercises:

Solve the following:

1. 7^4

2. 10^9

3. 1956^0

4. 83^2

5. 9^1

Multiply or divide to solve the following:

6. $3^2\cdot 3^5$

7. $h^6\cdot h$

8. $\frac{x^4y^4}{y^2x^3}$ (Hint: Take care of each variable separately.)

9. $(y^7)^9$

10. $(\frac{x^3x^5}{x^4x^2})^8$ (Hint: Take care of INSIDE the parenthesis first.)

Answer Key:

1. $7^4 \rightarrow 7\cdot7\cdot7\cdot7 = 2401$

2. $10^9 \rightarrow$ 1,000,000,000 (Remember the trick to this one is to make sure there are the number of zeros indicated by the exponent. In this case, there should be 9 zeros.)

3. $1956^0 \rightarrow 1$ (Remember that any number with a 0 exponent equals 1.)

4. $83^2 \rightarrow 83\cdot83 = 6889$

5. $9^1 \rightarrow 9$ (Any number with a 1 exponent is itself.)

Multiply or divide to solve

6. $3^2\cdot3^5 \rightarrow 3^{(2+5)} = 3^7 = 3\cdot3\cdot3\cdot3\cdot3\cdot3\cdot3 = 2187$

7. $h^6\cdot h \rightarrow h^{(6+1)} = h^7$

8. $\frac{x^4y^4}{y^2x^3} \rightarrow \frac{\cancel{x}\cdot\cancel{x}\cdot\cancel{x}\cdot x\cdot\cancel{y}\cdot\cancel{y}\cdot y\cdot y}{\cancel{y}\cdot\cancel{y}\cdot\cancel{x}\cdot\cancel{x}\cdot\cancel{x}} = xy^2$

9. $(y^7)^9 \rightarrow y^{7\cdot9}=y^{63}$

10. $(\frac{x^3x^5}{x^4x^2})^8 \rightarrow (\frac{\cancel{x}\cdot\cancel{x}\cdot\cancel{x}\cdot\cancel{x}\cdot\cancel{x}\cdot\cancel{x}\cdot x\cdot x}{\cancel{x}\cdot\cancel{x}\cdot\cancel{x}\cdot\cancel{x}\cdot\cancel{x}\cdot\cancel{x}})^8 = (x^2)^8 = x^{16}$

If you still have questions about working with exponents and are a UHV student, please come to the Academic Center for face-to-face tutoring. Academic Center hours can be found at www.uhv.edu/ac/tutoring/subject.aspx.

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